How to Fit the Data to a 2nd Degree Polynomial

The first step is to generate three equations in three unknowns by plugging the points (2, 469), (3,615), and and (4, 1,058) into the equation $y = ax^2 + bx + c$:

$a(2^2) + 2b + c = 469$
$a(3^2) + 3b + c = 615$
$a(4^2) + 4b + c = 1{,}058$

Simplifying, we get:

$4a + 2b + c = 469$
$9a + 3b + c = 615$
$16a + 4b + c = 1{,}058$

We can eliminate the c variable by subtracting equation 1 from equation 2. Similary, we can eliminate the c variable by subtracting equation 2 from equation 3. This yields two equations in two unknowns:

$5a + b = 146$
$7a + b = 443$

We can now eliminate the b variable by subtracting new equation 1 from new equation 2:

$2a = 297$

Dividing both sides by 2 yields:

$a = 148.5$

Then we can plug a = 148.5 into one of the equations in two unknowns to solve for b:

$(5)(148.5) + b = 146$
$742.5 + b = 146$
$b = 146 - 742.5$
$b = -596.5$

Then we can plug a = 148.5 and b = -596.5 into one of the three original equations and solve for c:

$(4)(148.5) + (2)(-596.5) + c = 469$
$594 - 1{,}193 + c = 469$
$-599 + c = 469$
$c = 469 + 599$
$c = 1{,}068$

Thus, the polynomial equation that passes through the points is: $y = 148.5x^2 - 596.5x + 1{,}068$.

An alternative approach is to represent the system of equations in matrix form and use Excel to do the heavy lifting.

$$\begin{vmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 469 \\ 615 \\ 1{,}058 \end{vmatrix}$$

We can then use the MINVERSE function in Excel to find the inverse of the 3x3 matrix shown above:

$$\begin{vmatrix} 0.5 & -1 & 0.5 \\ -3.5 & 6 & -2.5 \\ 6 & -8 & 3 \end{vmatrix}$$

This saves time because finding the matrix inverse using algebra is time-consuming. In fact, solving the matrix problem by hand is probably more time-consuming than solving the system of equations.

Now, we can rearrange the equation so that the [a,b,c] vector is equal to the inverse matrix shown above times the [469,615,1058] vector:

$$\begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 0.5 & -1 & 0.5 \\ -3.5 & 6 & -2.5 \\ 6 & -8 & 3 \end{vmatrix} \begin{vmatrix} 469 \\ 615 \\ 1{,}058 \end{vmatrix} = \begin{vmatrix} 148.5 \\ -596.5 \\ 1{,}068 \end{vmatrix}$$

Note the matrix multiplication involves multiplying each element in the matrix horizontally by each element in the vector vertically, and then adding. For example, 148.5 is determined as:

$$(0.5)(469) + (-1)(615) + (0.5)(1{,}058) = 234.5 - 615 + 529 = 148.5$$

Note the values of [a,b,c] = [148.5, -596.5, 1,068] are the same answers as solving the system of equations.

Press the back button to return to the blog entry. If you press the x, you'll have to start over.